Imprints of the Metrically-coupled Dilaton on Density Perturbations in Inflationary Cosmology

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Abstract

Spectra of density perturbations produced during chaotic inflation are calculated, taking both adiabatic and isocurvature modes into account in a class of scalar-tensor theories of gravity in which the dilaton is metrically coupled. Comparing the predicted spectrum of the cosmic microwave background radiation anisotropies with the one observed by the COBE-DMR we calculate constraints on the parameters of these theories, which turn out to be stronger by an order-of-magnitude than those obtained from post-Newtonian experiments.

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I. INTRODUCTION

Inflationary cosmology gives a natural explanation for the horizon, flatness, homogeneity, and monopole problems [1]. In its simplest form, inflation with a single scalar field predicts an approximately scale-invariant spectrum of Gaussian adiabatic density perturbations [2]. If other fields are present, however, primordially isocurvature perturbations may also be generated to leave an imprint on the universe today [3,4].

Scalar-tensor theories of gravitation have recently been received a renewed interest. The main reason is that the low-energy effective action of superstring theories [5] generally involves a dilaton coupled to the Ricci curvature. Moreover, as pointed out by Damour and Esposite-Farese [6] in the study of neutron star models, a wide class of scalar-tensor theories not only pass the present weak-field gravitational tests but also exhibit nonperturbative strong-field deviations away from general relativity. Detectability of scalar gravitational waves by laser interferometric gravitational wave observatories(LIGO) has been explored in [7,8].

Cosmological consequences of such theories, however, have not been fully examined. Recently, one of us studied the generality of chaotic inflation in scalar-tensor theories in detail and found that the onset of inflation could be greatly affected by the presence of the dilaton in some cases [9]. According to the attractor mechanism of scalar-tensor theories [10], which works only for those theories in which the derivative of the Brans-Dicke coupling is positive, even those theories that are quite different from general relativity in the early stage of the universe approach general relativity during the inflationary stage and the matter-dominated stage of the universe. Even if this is the case, the deviations in the earlier stage may be imprinted in the spectrum of the cosmic microwave background radiation (CMB). We may constrain model parameters using the CMB anisotropy as a probe of the early universe. In fact, because of the presence of two scalar fields, *i.e.*, the inflaton and the dilaton, density perturbations have both adiabatic and isocurvature modes and hence the spectrum may be changed.

In this paper, we study formation and evolution of density perturbations during and after chaotic inflation paying attention to the role of the isocurvature mode in a class of scalar-tensor theories in which the dilaton is metrically coupled. This paper originates from the analysis by Starobinsky and Yokoyama [11], who calculated the spectrum of density perturbations from inflation in Brans-Dicke theory. The scalar-tensor theories we study include those with negative β (the derivative of the Brans-Dicke coupling) which have not been studied in the literature [12] in which the numerical analysis is limited to positive β. These theories exhibit a significant deviation from general relativity in the strong field region [6].

The plan of this paper is as follows: In §2, basic equations of background fields are given. Linear perturbation equations are given and solutions to these equations are derived in §3. We study the evolution of isocurvature perturbations after inflation in §4. In §5, the spectrum of density perturbations are calculated and compared with the COBE observation to constrain the parameters of the theory. Finally §6 is devoted to summary.

II. BACKGROUND FIELD EQUATIONS

A. scalar-tensor theories of gravitation

We consider the inflaton, σ, coupled to scalar-tensor theories of gravity. The action is

$$S = \int d^4x \sqrt{-\bar{g}} \left[\frac{1}{16\pi} \left(\psi \bar{\mathcal{R}} - \frac{\omega(\psi)}{\psi} \bar{g}^{ab} \psi_{,a} \psi_{,b} \right) - \frac{1}{2} \bar{g}^{ab} \sigma_{,a} \sigma_{,b} - V(\sigma) \right], \tag{2.1}$$

where ψ is the massless Brans-Dicke dilaton, $\bar{\mathcal{R}}$ is the scalar curvature, $\omega(\psi)$ is a dimensionless coupling parameter and $V(\sigma)$ is the potential of the inflaton. Let us consider a conformal transformation,

$$\bar{g}_{ab} = \frac{1}{\psi G} g_{ab} \equiv e^{2a(\psi)} g_{ab}, \tag{2.2}$$

with $\psi = G^{-1}$ or $a(\psi) = 0$ today. Then the action can be written as

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \mathcal{R} - \frac{1}{2} g^{ab} \varphi_{,a} \varphi_{,b} - \frac{1}{2} e^{2a} g^{ab} \sigma_{,a} \sigma_{,b} - e^{4a} V(\sigma) \right], \tag{2.3}$$

where $\kappa^2 \equiv 8\pi G$ and φ is defined by

$$\varphi \equiv \frac{1}{\kappa} \int \frac{d\psi}{\psi} \sqrt{\omega(\psi) + \frac{3}{2}}. \tag{2.4}$$

We also define

$$\alpha(\varphi) \equiv \frac{1}{\kappa} \frac{da}{d\varphi}, \qquad \beta(\varphi) \equiv \frac{1}{\kappa} \frac{d\alpha}{d\varphi}, \tag{2.5}$$

which are related with the PPN parameters $(\gamma_{Edd}, \beta_{Edd})$ and are given as

$$|\gamma_{Edd} - 1| = \frac{4\alpha(\varphi)^2}{1 + 2\alpha(\varphi)^2} \bigg|_0, \quad |\beta_{Edd} - 1| = \frac{2\alpha(\varphi)^2 |\beta(\varphi)|}{1 + 2\alpha(\varphi)^2} \bigg|_0, \tag{2.6}$$

where the subscript 0 indicates the present value. Here the Brans-Dicke theory corresponds to $\beta(\varphi) = 0$ and the general relativity to $\alpha(\varphi) = 0$. The recent analyses of the experimental data yield [14]

$$|\gamma_{Edd} - 1| < 2 \times 10^{-3}, \quad |4\beta_{Edd} - \gamma_{Edd} - 3| < 1 \times 10^{-3}. \tag{2.7}$$

Combining them yields the following constraint on β_{Edd}

$$|\beta_{Edd} - 1| < 6 \times 10^{-4}. \tag{2.8}$$

B. background equations

We consider the spatially flat FRW spacetime as a background metric

$$ds^2 = -dt^2 + R(t)^2 d\mathbf{x}^2. \tag{2.9}$$

The equations of motion derived from the action Eq.(2.3) are

$$\left(\frac{\dot{R}}{R}\right)^2 \equiv H^2 = \frac{\kappa^2}{6}\left[\dot{\varphi}^2 + e^{2a}\dot{\sigma}^2 + 2e^{4a}V(\sigma)\right], \tag{2.10}$$

$$\ddot{\varphi} + 3H\dot{\varphi} = \kappa\alpha\left[e^{2a}\dot{\sigma}^2 - 4e^{4a}V(\sigma)\right], \tag{2.11}$$

$$\ddot{\sigma} + 3H\dot{\sigma} = -e^{2a}V'(\sigma) - 2\kappa\alpha\dot{\varphi}\dot{\sigma}, \tag{2.12}$$

$$\dot{H} = -\frac{\kappa^2}{2}(\dot{\varphi}^2 + e^{2a}\dot{\sigma}^2), \tag{2.13}$$

where $\dot{}$ and $'$ denote time and σ derivatives, respectively.

The conditions for slow-roll inflation ($|\ddot{\sigma}| \lesssim |3H\dot{\sigma}|, |\ddot{\varphi}| \lesssim |3H\dot{\varphi}|$) are written as [11–13],

$$\max\left(\frac{1}{2}\dot{\varphi}^2, \frac{1}{2}e^{2a}\dot{\sigma}^2\right) \lesssim e^{4a}V(\sigma), \tag{2.14}$$

$$e^{-2a}V'(\sigma)^2 \lesssim 6\kappa^2 V(\sigma)^2, \tag{2.15}$$

$$e^{-2a}V''(\sigma) \lesssim 3\kappa^2 V(\sigma), \tag{2.16}$$

$$8\alpha(\varphi)^2 \lesssim 3, \tag{2.17}$$

$$|16\alpha(\varphi)^2 + 4\beta(\varphi)| \lesssim 3. \tag{2.18}$$

When the above inequalities apply, field equations are simplified to

$$3H^2 \simeq \kappa^2 e^{4a}V(\sigma), \tag{2.19}$$

$$3H\dot{\varphi} \simeq -4\alpha\kappa e^{4a}V(\sigma), \tag{2.20}$$

$$3H\dot{\sigma} \simeq -e^{2a}V'(\sigma). \tag{2.21}$$

III. LINEAR PERTURBATIONS

A. basic equations

Now we turn to linear perturbations with a perturbed metric in the longitudinal gauge

$$ds^2 = -(1 + 2\Phi)dt^2 + R^2(1 - 2\Psi)d\mathbf{x}^2. \tag{3.1}$$

From the perturbed Einstein equations, each Fourier mode[1] satisfies the following equations of motion to first order [11,12],

---

[1] Our convention of the Fourier transformation is $Y(\mathbf{k}) = (2\pi)^{-3/2} \int d^3x e^{-i\mathbf{k}\mathbf{x}} Y(\mathbf{x})$ for each variable Y.

$$\Phi = \Psi, \tag{3.2}$$

$$\ddot{\delta\varphi} + 3H\dot{\delta\varphi} + \frac{k^2}{R^2}\delta\varphi + 4e^{4a}\kappa^2 V(\sigma)(4\alpha^2 + \beta)\delta\varphi - e^{2a}\kappa^2\dot{\sigma}^2(2\alpha^2 + \beta)\delta\varphi$$
$$= 4\dot{\Phi}\dot{\varphi} + 2e^{2a}\alpha\kappa\dot{\sigma}\dot{\delta\sigma} - 4e^{4a}\alpha\kappa(V'(\sigma)\delta\sigma + 2V(\sigma)\Phi), \tag{3.3}$$

$$\ddot{\delta\sigma} + 3H\dot{\delta\sigma} + \frac{k^2}{R^2}\delta\sigma + e^{2a}V''(\sigma)\delta\sigma = -2e^{2a}V'(\sigma)(\Phi + \alpha\kappa\delta\varphi)$$
$$+ \dot{\sigma}(4\dot{\Phi} - 2\alpha\kappa\dot{\delta\varphi} - 2\beta\kappa^2\dot{\varphi}\delta\varphi) - 2\alpha\kappa\dot{\varphi}\dot{\delta\sigma}, \tag{3.4}$$

$$\ddot{\Phi} + 4H\dot{\Phi} + (\dot{H} + 3H^2)\Phi = \frac{\kappa^2}{2}\left[\dot{\varphi}\dot{\delta\varphi} + e^{2a}\dot{\sigma}\dot{\delta\sigma} - e^{4a}V'(\sigma)\delta\sigma + (e^{2a}\dot{\sigma}^2 - 4e^{4a}V(\sigma))\alpha\kappa\delta\varphi\right], \tag{3.5}$$

together with the Hamiltonian and momentum constraints

$$3H\dot{\Phi} + (\dot{H} + 3H^2)\Phi + \frac{k^2}{R^2}\Phi = -\frac{\kappa^2}{2}\left[\dot{\varphi}\dot{\delta\varphi} + e^{2a}\dot{\sigma}\dot{\delta\sigma} + e^{4a}V'(\sigma)\delta\sigma + (e^{2a}\dot{\sigma}^2 + 4e^{4a}V(\sigma))\alpha\kappa\delta\varphi\right], \tag{3.6}$$

$$\dot{\Phi} + H\Phi = \frac{\kappa^2}{2}(\dot{\varphi}\delta\varphi + e^{2a}\dot{\sigma}\delta\sigma). \tag{3.7}$$

B. long wavelength perturbations

Since what we need are the non-decreasing adiabatic and isocurvature modes on large scale $k \ll RH$, which turn out to be weakly time-dependent as will be seen in the final result [4], we may consistently neglect $\dot{\Phi}$ and those terms containing two time derivatives. Then eqs. (3.3), (3.4), and (3.7) are simplified to

$$\Phi \simeq -2\alpha\kappa\delta\varphi - \frac{1}{2}\frac{V'}{V}\delta\sigma, \tag{3.8}$$

$$3H\dot{\delta\varphi} \simeq -4\beta\kappa^2 e^{4a}V\delta\varphi, \tag{3.9}$$

$$3H\dot{\delta\sigma} \simeq -e^{2a}\left(\frac{V'}{V}\right)' V\delta\sigma + 2\alpha\kappa e^{2a}V'\delta\varphi. \tag{3.10}$$

Following [11], we find the last two equations can be integrated to give

$$\delta\varphi \simeq \frac{4\alpha}{\kappa}Q_1, \tag{3.11}$$

$$\delta\sigma \simeq \frac{1}{\kappa^2}\frac{V'}{V}(e^{-2a}Q_1 + Q_2), \tag{3.12}$$

where use has been made of Eqs.(2.19), (2.20) and (2.21). We then find

$$\Phi \simeq -8\alpha^2 Q_1 - \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2 (e^{-2a}Q_1 + Q_2), \tag{3.13}$$

where Q_1 and Q_2 are constants of integration.

C. adiabatic and isocurvature modes

In order to clarify the physical meaning of the above solutions, we should divide them into adiabatic and isocurvature modes [11] [4]. The primordially isocurvature mode is characterized by its vanishingly small contribution to the curvature perturbation initially, while the growing adiabatic mode can be described by the following expressions in the long wavelength limit $k \ll RH$ (see Appendix A for a review of its derivation).

$$\Phi_{ad} = C_1 \left[1 - \frac{H}{R} \int R(t')dt' \right] \simeq -C_1 \frac{\dot{H}}{H^2}, \tag{3.14}$$

$$\frac{\delta \varphi_{ad}}{\dot{\varphi}} = \frac{\delta \sigma_{ad}}{\dot{\sigma}} = \frac{C_1}{R} \int R(t')dt' \simeq \frac{C_1}{H}, \tag{3.15}$$

as a solution of a second-order differential equation Eq.(A1) without the source term. Here C_1 is a constant and the latter approximate equality in each expression is derived by integration by parts during the inflationary stage.

As discussed by Starobinsky and Yokoyama [11] when the conditions

$$8\alpha^2 \ll \frac{1}{2\kappa^2} \left(\frac{V'}{V} \right)^2 e^{-2a}, \tag{3.16}$$

$$e^{-2a} \simeq 1 \tag{3.17}$$

hold, one can discriminate between adiabatic and isocurvature modes in the final results by defining new constants C_1 and C_3 as $C_1 \equiv -Q_1 - Q_2$ and $C_3 \equiv -Q_2$. Using Eqs.(2.19), (2.20) and (2.21) we find

$$\Phi \simeq -C_1 \frac{\dot{H}}{H^2} + C_3 \left[-8\alpha^2 + \frac{1}{2\kappa^2} \left(\frac{V'}{V} \right)^2 (1 - e^{-2a}) \right], \tag{3.18}$$

$$\frac{\delta \varphi}{\dot{\varphi}} \simeq \frac{C_1}{H} - \frac{C_3}{H}, \tag{3.19}$$

$$\frac{\delta \sigma}{\dot{\sigma}} \simeq \frac{C_1}{H} - \frac{C_3}{H} (1 - e^{2a}). \tag{3.20}$$

In the above expressions, terms in proportion to C_1 and C_3 represent adiabatic and isocurvature modes, respectively if the above conditions hold. In more general scalar-tensor theories (especially in positive β theories) Eq.(3.16) and Eq.(3.17) may not hold and consequently terms in proportion to C_3 contain the adiabatic mode partially. Then these terms might be better referred to as the non-adiabatic mode. But this is a matter of terminology that does not affect our final result and we keep using the same expression in this case, too.

D. quantum fluctuations

We shall next determine the constants C_1 and C_3 from amplitudes of quantum fluctuations of the scalar fields generated during the inflationary stage. Due to the inequalities (2.14),(2.15) and (2.16), Eqs.(3.3) and (3.4) can be approximated by the equation of motion

of a free massless scalar field during inflation for $k \geq RH$. The standard quantization gives the well-known result, that is, the Fourier components of the fields can be represented in the form

$$\delta\varphi(\mathbf{k}) = \frac{H(t_k)}{\sqrt{2k^3}}\epsilon_\varphi(\mathbf{k}), \ \ \delta\sigma(\mathbf{k}) = \frac{H(t_k)}{\sqrt{2k^3}}e^{-a}\epsilon_\sigma(\mathbf{k}), \tag{3.21}$$

where t_k is the time when k-mode leaves the Hubble horizon during inflation. Here $\epsilon_\varphi(k)$ and $\epsilon_\sigma(k)$ are classical random Gaussian variables with the following averages:

$$\langle\epsilon_\varphi(\mathbf{k})\rangle = \langle\epsilon_\sigma(\mathbf{k})\rangle = 0, \ \ \langle\epsilon_i(\mathbf{k})\epsilon_j(\mathbf{k}')\rangle = \delta_{ij}\delta^{(3)}(\mathbf{k} - \mathbf{k}'), \ \ i, j = \varphi, \sigma. \tag{3.22}$$

We thus find

$$C_1 = \left[e^{-2a}H\frac{\delta\sigma}{\dot{\sigma}} + (1 - e^{-2a})H\frac{\delta\varphi}{\dot{\varphi}}\right]_{t_k} = \frac{H^2(t_k)}{\sqrt{2k^3}}\left[\frac{e^{-3a}}{\dot{\sigma}}\epsilon_\sigma(\mathbf{k}) + \frac{1 - e^{-2a}}{\dot{\varphi}}\epsilon_\varphi(\mathbf{k})\right]_{t_k}, \tag{3.23}$$

$$C_3 = \left[e^{-2a}H\left(\frac{\delta\sigma}{\dot{\sigma}} - \frac{\delta\varphi}{\dot{\varphi}}\right)\right]_{t_k} = \frac{H^2(t_k)}{\sqrt{2k^3}}\left[\frac{e^{-3a}}{\dot{\sigma}}\epsilon_\sigma(\mathbf{k}) - \frac{e^{-2a}}{\dot{\varphi}}\epsilon_\varphi(\mathbf{k})\right]_{t_k}. \tag{3.24}$$

Note that in Ref. [12], they ignored the entropy perturbations at and after the end of inflation which corresponds to setting $C_3 = 0$ when they calculated the spectral index although they originally took into account these quantities in their derivation. This cannot be generally justified since both $\delta\sigma$ and $\delta\varphi$ are stochastic quantities.

IV. EVOLUTION OF ISOCURVATURE PERTURBATION AFTER INFLATION

In order to relate the spectrum of Φ at the end of inflation to that at decoupling, we have to study the evolution of the isocurvature mode together with the adiabatic mode after inflation. Unlike the adiabatic modes we do not have a universal formula for the isocurvature modes, so we have to calculate them explicitly.

A. isocurvature perturbation in the reheating stage with Brans-Dicke dilaton

First let us consider the isocurvature mode during the reheating stage after inflation. In this stage, $\kappa^2 V \ll e^{-2a}V''$ (see Eq.(2.16)), so that the inflaton oscillates coherently. Since equations of motion of perturbed quantities, (3.3) through (3.5), have oscillating coefficients, we must worry about possible parametric resonance effect. This issue has been studied by Kodama and Hamazaki [15] recently in the case only the inflaton is present and oscillating. They have shown that resonance effect is unimportant at least in the long wavelength regime which is of our interest. In the present case with two scalar fields, however, their approach is not directly applicable, in particular, for the isocurvature mode [16]. Hence we have numerically solved the evolution equations in the long wavelength limit. Figure 1 illustrates an example of the results for the Brans-Dicke theory with $\omega = 500$ and $\omega = 5$. We find that there is no anomalous growth of perturbation variables and that linear analysis suffices even in this regime. We also find qualitatively the same results for more generic scalar-tensor theories.

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B. Isocurvature perturbation in the radiation-dominated stage with Brans-Dicke dilaton

Next let us consider the isocurvature mode during the radiation dominated era after the reheating stage. In appendix B, a general expression of the evolution equation for Φ is given in the dilaton-inflaton-radiation system. After the inflaton has decayed into radiation, Eq.(B37) becomes

$$
\ddot{\Phi} + (4 + 3c_s^2)H\dot{\Phi} + (2\dot{H} + 3(1 + c_s^2)H^2)\Phi + c_s^2\frac{k^2}{R^2}\Phi
$$
$$
= \frac{\kappa^2}{3}\frac{h_r h_d}{h}\left(\frac{\Delta_d}{1 + w_d} - \frac{\Delta_r}{1 + w_r}\right) \tag{4.1}
$$

where c_s is the sound velocity,

$$
c_s^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{3\dot{\varphi}^2 + 4\rho_r/3}{3\dot{\varphi}^2 + 4\rho_r}, \tag{4.2}
$$

with $h_j \equiv \rho_j + p_j$, $w_j \equiv p_j/\rho_j$ $(j = r, d)$, and $h \equiv \rho + p$. It is now apparent that terms in the parenthesis in the right hand side of (4.1) are just the entropy perturbation

$$
\frac{\delta S}{S} = \frac{3}{4}\Delta_r - \frac{1}{2}\Delta_d. \tag{4.3}
$$

For the isocurvature perturbation, it is nonvanishing. It is known that if we replace ρ_d with the baryon density ρ_b, the isocurvature mode grows as R [17]. Because the dilaton density ρ_d decays as R^{-6} instead of R^{-3}, it is clear that the special solution originating from the source term decays as R^{-2}. On the other hand, the homogeneous part of the growing mode solution is the same as the adiabatic one. Therefore the isocurvature mode in the radiation dominated stage grows no faster than the adiabatic mode. This behavior can be also seen by solving the Eq.(B27) by using the Green's function method.

To conclude, the isocurvature mode does not evolve faster than the adiabatic counterpart and so is unimportant. The spectrum at the end of inflation is sufficient to compare the observation.

V. SPECTRA OF PERTURBATIONS AND CONSTRAINTS ON PPN PARAMETERS

Now we calculate the density fluctuation spectra produced by inflation. In order to do so, we have to specify $V(\sigma)$ and $a(\varphi)$. We consider chaotic inflation induced by a mass term $V(\sigma) = m^2\sigma^2/2$, of which we have a sensible particle-physics model in the intermediate scale [18]. Here m is determined from the large-angle microwave anisotropies seen by COBE-DMR [19].

A. general remark on initial conditions

Initial conditions for φ and σ are given in terms of the number of e-foldings from the end of inflation;

$$N_I = -\int_{t_e} H dt \simeq \frac{\kappa}{4} \int_{\varphi_e} \frac{d\varphi}{\alpha(\varphi)} \simeq \kappa^2 \int_{\sigma_e} \frac{e^{2a} V(\sigma)}{V'(\sigma)} d\sigma, \tag{5.1}$$

where the suffix e represents the end epoch of inflation. Our present horizon ($H_0^{-1} = 3000 h^{-1}$Mpc) crossed outside the Hubble scale about 60 e-foldings before the end of inflation. We assume that the slow-roll condition is satisfied at N_I e-folds from the end of inflation. Then the terms containing the two time derivative can be neglected in Eqs.(3.3-3.7). Initial conditions for $H, \dot{\varphi}, \dot{\sigma}$ are given by Eqs.(2.19-2.21).

We need to determine φ_e to calculate the spectrum of density perturbations. We normalize φ in terms of its present value so that $\alpha^2(\varphi) = \alpha_0^2$ (see (2.6)) at present. Then we are able to specify φ_e by going back to the end of inflation.

In order to connect with the present length scale, we have to estimate the number of e-foldings from the end of inflation to the present [20]. After the end of inflation, the oscillating inflaton, which behaves like a dust fluid for the massive inflaton, dominates the universe, and then radiation dominated stage follows. The effective temperature, $T_{\rm rd}$, and the scale factor $R_{\rm rd}$, at the onset of radiation dominated stage is determined from the energy density

$$\rho_e \left(\frac{R_e}{R_{\rm rd}}\right)^3 = g_{\rm eff} \frac{\pi^2}{30} T_{\rm rd}^4, \tag{5.2}$$

where ρ_e is the energy density at the end of inflation and $g_{\rm eff}$ is the effective number of degrees of freedom at temperature T typically taking the value of $O(100)$ at $T_{\rm rd}$. Assuming the conservation of the entropy in relativistic particles per comoving volume $s = \frac{4}{3} R^3 \rho/T$, we have

$$R_{\rm rd}^3 g_{\rm eff}(T_{\rm rd}) T_{\rm rd}^3 = (2 T_{\gamma 0}^3 + \frac{21}{4} T_{\nu 0}^3) R_0^3$$
$$= \frac{43}{11} T_{\gamma 0}^3 R_0^3, \tag{5.3}$$

where $T_{\nu 0}$ is the background neutrino temperature. The number of e-folding from the end of inflation to the present is then

$$N_e = \ln \frac{R_0}{R_e}$$
$$\simeq 72 + \frac{1}{3} \ln \frac{\rho_e}{m_{\rm pl}^4} - \frac{1}{3} \ln \frac{T_{\rm rd}}{m_{\rm pl}}, \tag{5.4}$$

where $m_{\rm pl}$ is the Planck mass. We set the present value of the scale factor R_0 to unity for convenience.

Given initial conditions, we solve Eqs.(2.10-2.13) numerically using a fourth-order Runge-Kutta method. We choose $N_I = 65$ typically. We express all quantities in units of $m_{\rm pl}$ and use $x = \ln R$ for the integration variable instead of the cosmic time [20].

The calculations of the perturbed equations Eqs.(3.3-3.7) are begun when $k \leq RH$. The initial conditions for $\delta\varphi$ and $\delta\sigma$ are given by Eq.(3.21). Since the background fields are in their slow-rolling phase, the initial conditions for $\dot{\delta\varphi}, \dot{\delta\sigma}, \Phi$, and $\dot{\Phi}$ are given by neglecting terms with the second order time derivative in Eqs.(3.3), (3.4), (3.6) and (3.7). We have to solve Eq.(A1) with or without the source term for the evolution of Φ so that we can follow the evolution of adiabatic and isocurvature modes separately. The spectral index n defined by

$$n - 1 = \frac{d \ln k^3 \langle |\Phi|^2 \rangle}{d \ln k} \tag{5.5}$$

is calculated for the present horizon scale.

B. COBE normalization

We have to normalize fluctuations to compare the spectra among different gravitational theories. The COBE-DMR data give the normalization. Following the standard treatment, the temperature fluctuations are expanded in terms of spherical harmonics as

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi), \tag{5.6}$$

and we define

$$\langle a_{lm}^* a_{l'm'} \rangle \equiv C_l \delta_{ll'} \delta_{mm'}. \tag{5.7}$$

On COBE-DMR scales, temperature fluctuations are dominated by the Sachs-Wolfe effect which is expressed as $\Delta T(x)/T = \Phi(x)/3$ [21] for scalar perturbations. Here we neglect acoustic oscillation terms which may contribute to higher l's. Therefore C_l is written as

$$C_l \equiv \langle |a_{lm}|^2 \rangle$$
$$= \frac{2}{\pi} \int dk k^2 \left\langle \left| \frac{\Phi}{3} \right|^2 \right\rangle j_l^2(k\eta_0), \tag{5.8}$$

where $\eta_0 = 2H_0^{-1}$ is the conformal time at present.

If we approximate the power spectrum of Φ by a power law,

$$k^3 \langle |\Phi|^2 \rangle = A \left(\frac{k}{H_0} \right)^{n-1}, \tag{5.9}$$

we can write Eq.(5.8) for scalar perturbations as

$$C_l = \frac{A}{36} \frac{\Gamma(3 - n)\Gamma(l + \frac{n-1}{2})}{\Gamma^2(\frac{4-n}{2})\Gamma(l + \frac{5-n}{2})}. \tag{5.10}$$

The rms quadrupole Q_{rms-PS} used by the COBE-DMR group is written in terms of C_2 as

$$Q_{rms-PS} = T_0 \sqrt{\frac{5C_2}{4\pi}}, \tag{5.11}$$

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where $T_0 = 2.726$K. According to the 4 yr COBE-DMR data [19]

$$Q_{rms-PS} = 18\mu\text{K}, \tag{5.12}$$

for $n = 1$ Harrison-Zel'dovich spectrum. It takes a slightly different value if different power low indices are considered, but we normalize the spectra by using the above value because the index is varied around $n = 1$ and furthermore the constraints on the scalar-tensor theories are imposed primarily from the spectral shape or its power-law index rather than from its amplitude. The corresponding amplitude A is

$$A = \frac{144\pi}{5} \frac{\Gamma^2(\frac{4-n}{2})\Gamma(l + \frac{5-n}{2})}{\Gamma(3-n)\Gamma(l + \frac{n-1}{2})} \left(\frac{Q_{rms-PS}}{T_0}\right)^2. \tag{5.13}$$

Eventually m takes the value ranging from $O(10^{-6}m_{\text{pl}})$ to $O(10^{-8}m_{\text{pl}})$.

C. Brans-Dicke theory

The Brans-Dicke theory corresponds to $\beta \equiv 0$ and thus

$$\alpha(\varphi) = \alpha_0, \tag{5.14}$$
$$a(\varphi) = \alpha_0\kappa\varphi. \tag{5.15}$$

Initial conditions for φ and σ are

$$\kappa\varphi = 4\alpha_0 N + \kappa\varphi_e, \tag{5.16}$$
$$\kappa\varphi_e \simeq 10\alpha_0, \tag{5.17}$$
$$\kappa^2\sigma^2 = \frac{2}{3}e^{-2a(\varphi_e)} - \frac{1}{2\alpha_0^2}(e^{-2a(\varphi)} - e^{-2a(\varphi_e)}). \tag{5.18}$$

In Figs. 2, the spectra $k^3\langle|\Phi|^2\rangle$ for each α_0 or ω are shown. In the Brans-Dicke theory, the conditions Eqs.(2.14) and (2.15) hold and therefore we are justified to say that terms in proportion to C_3 represent the isocurvature mode. We see that considering the current limit on ω ($\omega > 500$) in Eq.(2.18), the contribution of the isocurvature mode is totally negligible in the Brans-Dicke theory in agreement with Starobinsky and Yokoyama [11].

The spectral indices on the comoving horizon scale today are $n = 0.962, 0.966, 0.967$ for $\omega = 500, 5000, 50000$, respectively. Note that $n = 0.967$ in general relativity.

D. a class of scalar-tensor theory

We take the following simple functional form for $a(\varphi)$ in accord with Damour and Nordtvedt [10]

$$a(\varphi) = \frac{\beta}{2}\kappa^2(\varphi^2 - \varphi_0^2), \qquad \beta \equiv \text{const}, \tag{5.19}$$
$$\alpha(\varphi) = \beta\kappa\varphi. \tag{5.20}$$

We choose the origin of the field φ so that $a(\varphi) = 0$ at present in order to reproduce the correct value of the gravitational constant.[2] We have to determine the value of φ at the end of inflation, *i.e.*, φ_e, to calculate the spectrum of density perturbation. Since we normalize φ in terms of its present value, we need to go back to the end of inflation to specify φ_e. The evolution of φ in the radiation or matter-dominated universe is analyzed by Damour and Nordtvedt [10](see also [22]). In the radiation-dominated stage φ is hardly changed. We assume for simplicity that φ_e is equal to φ at the beginning of matter dominated stage. For $|\beta| \ll 1$(see Eq.(2.18)), φ_e is determined by

$$\kappa\varphi_e \simeq \frac{\alpha_0}{\beta}e^{10\beta}. \tag{5.21}$$

Then φ is given in terms of N as

$$\kappa\varphi \simeq \kappa\varphi_e e^{4\beta N} \simeq \frac{\alpha_0}{\beta}e^{4\beta N+10\beta}. \tag{5.22}$$

For σ with (see Eq.(2.15))

$$e^{-2a(\varphi_e)}\left(\frac{V'(\sigma_e)}{\kappa V(\sigma_e)}\right)^2 = 6, \tag{5.23}$$

σ is given by

$$\kappa^2\sigma^2 \simeq \frac{2}{3}e^{-2a(\varphi_e)} + \frac{e^{\alpha_0^2/\beta}}{2\beta}\left[E_i(-\beta(\kappa\varphi)^2) - E_i(-\beta(\kappa\varphi_e)^2)\right], \tag{5.24}$$

where $E_i(x)$ is the exponential integral function defined by $E_i(-x) = -\int_x^\infty e^{-t}dt/t$.

In Figs. 3, the spectra for several β with $\alpha_0 = 10^{-3}$ are shown. In some cases the mode coming from C_3 part in Eq.(3.18) is not negligible and the spectrum can be different from flat one. In scalar-tensor theories with positive β, the conditions Eqs.(3.16) and (3.17) do not hold since initially α can be greatly different from zero (see Eq.(5.22)), and terms in proportion to C_3 contain the adiabatic mode in addition to the isocurvature mode. For theories with negative β, however, terms in proportion to C_3 represent the isocurvature mode.

In Fig. 4 we show that contour plot of n on $\alpha_0 - \beta$ plane. We see that for $\beta > 0$ n falls down very steeply with increasing β. Thus the observed spectrum by COBE-DMR [19], $n = 1.2 \pm 0.3$, strongly constrains β. This possibility was pointed out in [12] for $\beta > 0$. Interestingly, the contour level can be fitted by a linear function in $\log\alpha_0$. For $n \geq 0.9$,

$$9.0 \times 10^{-3}\log\left(\frac{\alpha_0}{\sqrt{5}\times 10^{-2}}\right) - 1.5 \times 10^{-2} \leq \beta \leq -8.9 \times 10^{-3}\log\left(\frac{\alpha_0}{\sqrt{5}\times 10^{-2}}\right) + 5.3 \times 10^{-3}. \tag{5.25}$$

---

[2] The authors of [12] take $\alpha(\phi) = a_1 + a_2\kappa\phi$ and choose the origin so that $\alpha = a_1$ at the end of inflation. However, α deviates from a_1 afterwards and may conflict with the limit on α_0.

For $n \geq 0.7$,

$$5.0 \times 10^{-3} \log \left(\frac{\alpha_0}{\sqrt{5 \times 10^{-2}}} \right) - 3.7 \times 10^{-2} \leq \beta \leq -9.2 \times 10^{-3} \log \left(\frac{\alpha_0}{\sqrt{5 \times 10^{-2}}} \right) + 7.3 \times 10^{-3}.$$

(5.26)

Since the tensor mode of perturbations does not contribute to the spectrum significantly in the above range of the spectral index, the above inequality gives the constraint on β. Note that the constraint on β has little dependence on α_0 as well as on the details of normalization of fluctuations; adopting the inflaton mass which is ten times larger results in only a few percent changes in the constraint. We also note that the constraint is stronger by an order-of-magnitude than those obtained by post-Newtonian experiments(see Eq.(2.8)).

VI. SUMMARY

We have studied density perturbations produced during chaotic inflation taking both adiabatic and isocurvature modes into account in a class of scalar-tensor theories of gravity in which the dilaton coupling is metric one. The spectrum of the density perturbation produced by chaotic inflation in a scalar-tensor theory can be a non-flat one because of the variable Brans-Dicke coupling. The spectrum observed by COBE-DMR thus constrains such a coupling. Assuming a simple coupling function, the constraint is found to be stronger by an order-of-magnitude than those obtained by post-Newtonian experiments. It is interesting to note that in the case of the simple coupling function $\alpha(\varphi) = \beta\varphi$ employed here, our constraint is much more stringent than that obtained by the binary pulsar experiment [23], $\beta > -5$, in a sense. Indeed α_0 must be $\alpha_0 < 10^{-1000}$(!) so that the constraint Eq.(5.25) or Eq.(5.26) is looser than $\beta > -5$ if these fitting formulas are extrapolated thus far. Thus we may conclude that cosmological inflation is suited in the framework of general relativity, or more advocatively, inflation favors Einstein.

ACKNOWLEDGMENTS

APPENDIX A: FORMULAE FOR ADIABATIC MODE

In this appendix we review derivation of the formulae for the adiabatic mode Eqs.(3.14) and (3.15). See [4] for a different approach.

Using Eqs.(3.5), (3.6) and (3.7), we get

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$$\ddot{\Phi} + (4 + 3c_s^2)H\dot{\Phi} + (2\dot{H} + 3(1 + c_s^2)H^2)\Phi + c_s^2\frac{k^2}{R^2}\Phi$$

$$= (c_s^2 - 1)\frac{k^2}{R^2}\Phi + \kappa^2\frac{e^{4a}}{\dot{\varphi}^2 + e^{2a}\dot{\sigma}^2}(V'\dot{\varphi} - 4e^{2a}\alpha\kappa V\dot{\sigma})(\dot{\sigma}\delta\varphi - \dot{\varphi}\delta\sigma), \quad \text{(A1)}$$

where c_s^2 is defined by

$$c_s^2 = \frac{\dot{p}}{\dot{\rho}} = 1 + \frac{2e^{4a}(4\alpha\kappa V\dot{\varphi} + V'\dot{\sigma})}{3H(\dot{\varphi}^2 + e^{2a}\dot{\sigma}^2)}. \quad \text{(A2)}$$

The left-hand-side of Eq.(A1) is just the same as that of the adiabatic perturbation for hydro-dynamical matter and in case of the adiabatic perturbation right-hand-side term vanishes. For long wavelength perturbations right-hand-side vanishes when

$$\frac{\delta\varphi}{\dot{\varphi}} = \frac{\delta\sigma}{\dot{\sigma}}. \quad \text{(A3)}$$

To derive the solution for the adiabatic mode Eq.(3.14), it is convenient to use the constancy of the Bardeen's ζ

$$\zeta \equiv -\frac{H^2}{\dot{H}}(\Phi + H^{-1}\dot{\Phi}) + \Phi = \text{const} \equiv C_1, \quad \text{(A4)}$$

which follows from the fact that the left-hand-side of Eq.(A1) is equal to $-\dot{H}\zeta/H$ in the long-wavelength limit. Substituting $\Phi - C_1 = \tilde{\Phi}$, the above equation becomes a homogeneous form

$$\dot{\tilde{\Phi}} + H\tilde{\Phi} - \frac{\dot{H}}{H}\tilde{\Phi} = -C_1 H. \quad \text{(A5)}$$

Noting the left-hand-side is written as

$$\frac{H}{R}\left(\frac{R}{H}\tilde{\Phi}\right)^{\cdot}, \quad \text{(A6)}$$

the solution is immediately given as

$$\tilde{\Phi} = -C_1\frac{H}{R}\int R(t')dt' + C_2\frac{H}{R}, \quad \text{(A7)}$$

where C_2 corresponds to the amplitude of the decaying mode which is neglected in Eq.(3.14).

To derive Eq.(3.15), we use Eq.(3.7) as

$$\dot{\Phi} + H\Phi = -C_1\frac{\dot{H}}{R}\int Rdt' + C_2\frac{\dot{H}}{R}$$

$$= \frac{\kappa^2}{2}(\dot{\varphi}\delta\varphi + e^{2a}\dot{\sigma}\delta\sigma) = \frac{\kappa^2}{2}\frac{\delta\varphi}{\dot{\varphi}}(\dot{\varphi}^2 + e^{2a}\dot{\sigma}^2). \quad \text{(A8)}$$

Employing Eq.(2.13) we find

$$\frac{\delta\varphi}{\dot{\varphi}} = \frac{\delta\sigma}{\dot{\sigma}} = \frac{1}{R}\left(C_1\int Rdt' - C_2\right). \quad \text{(A9)}$$

Neglecting the decaying mode, we get Eq.(3.15).

APPENDIX B: LINEAR PERTURBATIONS FOR DILATON-INFLATON-RADIATION SYSTEM

Basic equations in the Einstein frame are the Einstein equation

$$G_{ab} = \kappa^2 \left(T_{ab} + \partial_a \varphi \partial_b \varphi - \frac{1}{2} g_{ab} g^{cd} \partial_c \varphi \partial_d \varphi + e^{2a} \partial_a \sigma \partial_b \sigma - g_{ab} (\frac{1}{2} g^{cd} e^{2a} \partial_c \sigma \partial_d \sigma - e^{4a} V(\sigma)) \right),$$

(B1)

the dilaton equation of motion

$$\Box \varphi = -\kappa \alpha(\varphi)(T - g^{cd} e^{2a} \partial_c \sigma \partial_d \sigma + 4 e^{4a} V(\sigma)),$$

(B2)

the matter equation of motion (Bianchi identity)

$$\nabla_b T^b{}_a = \kappa \alpha (T - g^{cd} e^{2a} \partial_c \sigma \partial_d \sigma + 4 e^{4a} V(\sigma)) \nabla_a \varphi,$$

(B3)

and the inflaton equation of motion

$$\Box \sigma = e^{2a} V'(\sigma) - 2\kappa \alpha g^{ab} \varphi_{,a} \sigma_{,b},$$

(B4)

where $\kappa^2 = 8\pi G$.

1. background equations

The matter energy momentum tensor is given by

$$T_{ab} = \rho_r u_a u_b + p_r (g_{ab} + u_a u_b).$$

(B5)

Assuming the flat FRW model, the background equations are

$$H^2 = \frac{\kappa^2}{3} \left(\rho_r + \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} e^{2a} \dot{\sigma}^2 + e^{4a} V(\sigma) \right),$$

(B6)

$$\dot{H} = -\frac{\kappa^2}{2} \left(\rho_r + p_r + \dot{\varphi}^2 + e^{2a} \dot{\sigma}^2 \right),$$

(B7)

$$\ddot{\varphi} + 3H\dot{\varphi} = \kappa \alpha(-\rho_r + 3p_r + e^{2a} \dot{\sigma}^2 - 4 e^{4a} V(\sigma)),$$

(B8)

$$(\rho_r R^3)^{\cdot} + p_r (R^3)^{\cdot} = (\rho_r - 3p_r) R^3 \dot{a}(\varphi),$$

(B9)

$$\ddot{\sigma} + 3H\dot{\sigma} = -e^{2a} V'(\sigma) - 2\kappa \alpha \dot{\varphi} \dot{\sigma},$$

(B10)

where $p_r = \rho_r / 3$.

2. linear perturbations

We employ the metric perturbation in the longitudinal gauge as

$$ds^2 = -(1 + 2\Phi) dt^2 + R^2 (1 - 2\Phi) d\mathbf{x}^2.$$

(B11)

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Then the perturbed Einstein tensors are [24]

$$\delta G^t{}_t = \frac{2}{R^2}\left[3\dot{R}^2\Phi + 3R\dot{R}\dot{\Phi} + k^2\Phi\right], \tag{B12}$$

$$\delta G^t{}_j = \frac{2k}{R^2}\left[\dot{R}\Phi + R\dot{\Phi}\right]Y_j, \tag{B13}$$

$$\delta G^i{}_j = \frac{2}{R^2}\left[(\dot{R}^2 + 2R\ddot{R})\Phi + R\dot{R}\dot{\Phi} + (R^2\dot{\Phi})^{\cdot} + R\dot{R}\dot{\Phi}\right]\delta^i_j. \tag{B14}$$

As for the matter components, hydrodynamical perturbations of radiation are

$$\delta T^t{}_t = -\rho_r\delta, \tag{B15}$$
$$\delta T^t{}_j = (\rho_r + p_r)vY_j, \tag{B16}$$
$$\delta T^i{}_j = p_r\pi_L\delta^i_j. \tag{B17}$$

The dilaton perturbations are expressed as

$$\delta T^t{}_t = \dot{\varphi}^2\Phi - \dot{\varphi}\dot{\delta\varphi}, \tag{B18}$$

$$\delta T^t{}_j = \frac{k}{R}\dot{\varphi}\delta\varphi Y_j, \tag{B19}$$

$$\delta T^i{}_j = \left[-\dot{\varphi}^2\Phi + \dot{\varphi}\dot{\delta\varphi}\right]\delta^i_j. \tag{B20}$$

and the inflaton perturbations as

$$\delta T^t{}_t = -e^{2a}(\dot{\sigma}\dot{\delta\sigma} - \dot{\sigma}^2\Phi) - \kappa\alpha e^{2a}\dot{\sigma}^2\delta\varphi, \tag{B21}$$

$$\delta T^t{}_j = \frac{k}{R}e^{2a}\dot{\sigma}\delta\sigma Y_j, \tag{B22}$$

$$\delta T^i{}_j = \left[e^{2a}(\dot{\sigma}\dot{\delta\sigma} - \dot{\varphi}^2\Phi) + \kappa\alpha e^{2a}\dot{\sigma}^2\delta\varphi\right.$$
$$\left. - e^{4a}V'(\sigma)\delta\sigma - 4\kappa\alpha e^{4a}V(\sigma)\delta\varphi\right]\delta^i_j. \tag{B23}$$

Thus we have the perturbed Einstein equations

$$2\left[3\dot{H}\dot{\Phi} + 3H^2\Phi + \frac{k^2}{R^2}\Phi\right] = -\kappa^2\left[-(\dot{\varphi}^2 + e^{2a}\dot{\sigma}^2)\Phi + \rho_r\delta + \dot{\varphi}\delta\varphi + e^{2a}\dot{\sigma}\dot{\delta\sigma}\right.$$
$$\left. + \kappa\alpha e^{2a}\dot{\sigma}^2\delta\varphi + e^{4a}V'(\sigma)\delta\sigma + 4\kappa\alpha e^{4a}V(\sigma)\delta\varphi\right], \tag{B24}$$

$$2\left[\dot{\Phi} + H\Phi\right] = \kappa^2\left[(\rho_r + p_r)v\frac{R}{k} + \dot{\varphi}\delta\varphi + e^{2a}\dot{\sigma}\delta\sigma\right], \tag{B25}$$

$$2\left[\ddot{\Phi} + 4H\dot{\Phi} + (2\dot{H} + 3H^2)\Phi\right] = \kappa^2\left[-(\dot{\varphi}^2 + e^{2a}\dot{\sigma}^2)\Phi + \frac{1}{3}\rho_r\delta + \dot{\varphi}\dot{\delta\varphi} + e^{2a}\dot{\sigma}\dot{\delta\sigma}\right.$$
$$\left. + \kappa\alpha e^{2a}\dot{\sigma}^2\delta\varphi - e^{4a}V'(\sigma)\delta\sigma - 4\kappa\alpha e^{4a}V(\sigma)\delta\varphi\right]. \tag{B26}$$

The perturbed equations are rewritten as

$$\ddot{\Phi} + (4 + 3c_s^2)H\dot{\Phi} + (2\dot{H} + 3(1 + c_s^2)H^2)\Phi + c_s^2\frac{k^2}{R^2}\Phi$$
$$= (c_s^2 - 1)\frac{k^2}{R^2}\Phi - \frac{\kappa^2}{2}\left[\frac{2}{3}\rho_r\delta + 2e^{4a}V'(\sigma)\delta\sigma + 8\kappa\alpha e^{4a}V(\sigma)\delta\varphi\right.$$
$$\left. + \frac{1}{h}(2Hh_r - 2^{4a}V'(\sigma)\dot{\sigma} - 8\kappa\alpha e^{4a}V(\sigma)\dot{\varphi})(h_r\frac{vR}{k} + \dot{\varphi}\delta\varphi + e^{2a}\dot{\sigma}\delta\sigma)\right], \tag{B27}$$

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where c_s^2 is defined by

$$c_s^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{4H\rho_r/3 + 3H(\dot{\varphi}^2 + e^{2a}\dot{\sigma}^2) + 2e^{4a}V'(\sigma)\delta\varphi + 8\kappa\alpha e^{4a}V(\sigma)\delta\varphi}{4H\rho_r + 3H(\dot{\varphi}^2 + e^{2a}\dot{\sigma}^2)}. \tag{B28}$$

We have arranged the left-hand-side of Eq.(B27) so that it is just the same as that of the adiabatic perturbation for hydrodynamical matter.

Now let us examine the structure of the right-hand-side of Eq.(B27) in detail. Before doing so, we define the following useful gauge invariant variables [24]. For a general multi-component system (ρ_a, p_a), δ_a and v_a are defined by

$$\delta T^t{}_t = -\rho_a \delta_a, \tag{B29}$$

$$\delta T^t{}_j = h_a v_a Y_j, \tag{B30}$$

where $h_a = \rho_a + p_a$. The associated gauge invariant variables Δ_a and V_a are defined by

$$\Delta_a = \delta_a + 3(1 + w_a)(1 - q_a)\frac{RH}{k}V_T, \tag{B31}$$

$$V_a = v_a, \tag{B32}$$

where $w_a = p_a/\rho_a$ and $V_T = \Sigma h_a V_a/h$ with $h = \Sigma h_a$ and q_a is the source term for each energy momentum tensor. For the case we are considering, i.e., the radiation-inflaton-dilaton system, the respective $\Delta_a(\Delta_r, \Delta_i, \Delta_d)$ are written as

$$\frac{\Delta_r}{1 + w_r} = \frac{3}{4}\delta + 3\frac{RH}{k}\left[1 + \frac{(e^{2a}\dot{\sigma}^2 - 4e^{4a}V(\sigma))\kappa\alpha\dot{\varphi}}{3Hh_r}\right]V_T, \tag{B33}$$

$$\frac{\Delta_i}{1 + w_i} = \Phi + \frac{\dot{\delta\sigma}}{\dot{\sigma}} + \kappa\alpha\delta\varphi + 3\frac{RH}{k}V_T, \tag{B34}$$

$$\frac{\Delta_d}{1 + w_d} = \Phi + \frac{\dot{\delta\varphi}}{\dot{\varphi}} + 3\frac{RH}{k}\left[1 - \frac{(e^{2a}\dot{\sigma}^2 - 4e^{4a}V(\sigma))\kappa\alpha\dot{\varphi}}{3Hh_d}\right]V_T. \tag{B35}$$

Then Eq.(B27) can be rewritten as

$$\ddot{\Phi} + (4 + 3c_s^2)H\dot{\Phi} + (2\dot{H} + 3(1 + c_s^2)H^2)\Phi + c_s^2\frac{k^2}{R^2}\Phi$$

$$= (c_s^2 - 1)\frac{k^2}{R^2}\Phi - \frac{\kappa^2}{2}\left[\frac{1}{2}h_r\delta + 2h_r V_T\frac{RH}{k}\right.$$

$$+ 2\frac{h_r}{h}e^{2a}V'(\sigma)\dot{\sigma}\left(\frac{\delta\sigma}{\dot{\sigma}} - \frac{vR}{k}\right) + 8\frac{h_r}{h}\kappa\alpha e^{4a}V(\sigma)\dot{\varphi}\left(\frac{\delta\varphi}{\dot{\varphi}} - \frac{vR}{k}\right)$$

$$\left. + \frac{2e^{4a}\dot{\sigma}\dot{\varphi}}{h}(V'(\sigma)\dot{\varphi} - 4\kappa\alpha e^{2a}V(\sigma)\dot{\sigma})\left(\frac{\delta\sigma}{\dot{\sigma}} - \frac{\delta\varphi}{\dot{\varphi}}\right)\right]. \tag{B36}$$

When radiation is absent the expression agrees with that of inflaton-dilaton system.

By expanding out the term $(c_s^2 - 1)\frac{k^2}{R^2}\Phi$, we finally have

$$
\ddot{\Phi} + (4 + 3c_s^2)H\dot{\Phi} + (2\dot{H} + 3(1 + c_s^2)H^2)\Phi + c_s^2\frac{k^2}{R^2}\Phi
$$

$$
= -\frac{\kappa^2}{2}\left\{\frac{2}{3h}\left[h_r h_i\left(\frac{\Delta_r}{1 + w_r} - \frac{\Delta_i}{1 + w_i}\right) + h_r h_d\left(\frac{\Delta_r}{1 + w_r} - \frac{\Delta_d}{1 + w_d}\right)\right] - \frac{2RV_T}{3k}(\dot{\sigma}^2 - 4V(\sigma))\kappa\alpha\dot{\varphi}\right.
$$

$$
+ \frac{1}{3Hh}(2V'(\sigma)e^{4a}\dot{\sigma} + 8\kappa\alpha e^{4a}V(\sigma)\dot{\varphi})(\rho\Delta + e^{2a}V'(\sigma)\delta\sigma + 4\kappa\alpha e^{4a}V(\sigma)\delta\varphi)
$$

$$
+ 2\frac{h_r}{h}e^{2a}V'(\sigma)\dot{\sigma}\left(\frac{\delta\sigma}{3\dot{\sigma}} - \frac{vR}{k}\right) + 8\frac{h_r}{h}\kappa\alpha e^{4a}V(\sigma)\dot{\varphi}\left(\frac{\delta\varphi}{3\dot{\varphi}} - \frac{vR}{k}\right)
$$

$$
\left. + \frac{2e^{4a}\dot{\sigma}\dot{\varphi}}{h}(V'(\sigma)\dot{\varphi} - 4\kappa\alpha e^{2a}V(\sigma)\dot{\sigma})\left(\frac{\delta\sigma}{\dot{\sigma}} - \frac{\delta\varphi}{\dot{\varphi}}\right)\right\}. \tag{B37}
$$

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FIGURE CAPTION

Fig.1. Evolution of the inflaton and the curvature perturbations during the oscillation regime of the inflaton in the Brans-Dicke theory with (a)$\omega = 500$ and (b)$\omega = 5$. The abscissa is the e-folding number after the end of inflation.

Fig.2. The spectra of perturbations in Brans-Dicke theory with $\omega = 500, 5000, 50000$. The abscissa is the wavenumber in unit of $h\mathrm{Mpc}^{-1}$. Logarithm is base 10. The isocurvature mode is negligible in this theory.

Fig.3. The spectra of perturbations in scalar-tensor theories with $\alpha_0 = 1.0 \times 10^{-3}$ and $\beta = 0.02, 0.005, -0.04$. For larger $|\beta|$, the mode coming from C_3 part in Eq.(3.18) is not negligible and the spectrum has slope. The respective spectral index is $n = 0.640, 0.967, 0.734$.

Fig.4. The contour plot of spectral index n for scalar-tensor theories. The contour levels $n = 0.9, 0.7$ are shown. The region bounded by these courves is the allowed region. We find non-zero β theories are strongly constrained. The shaded region is excluded by the solar system experiments; the vertical line is the constraint by the expetiment of the deflection of light (or the time delay of light): $\alpha_0 < \sqrt{5} \times 10^{-2}$. The dotted curve in the shaded region is the constraint by the lunar-raser-ranging experiment: $\alpha_0^2|\beta_0| < 3 \times 10^{-4}$.
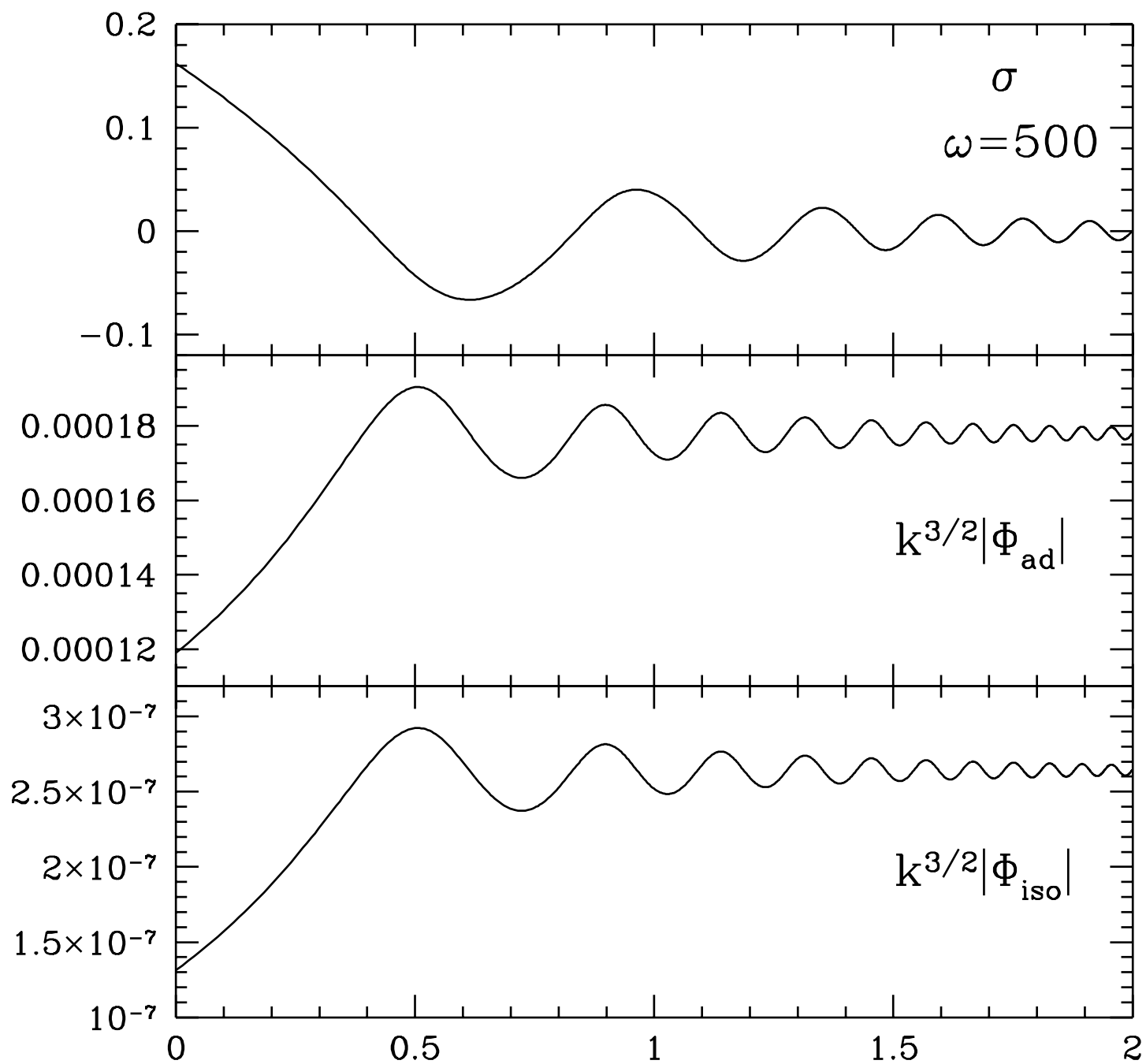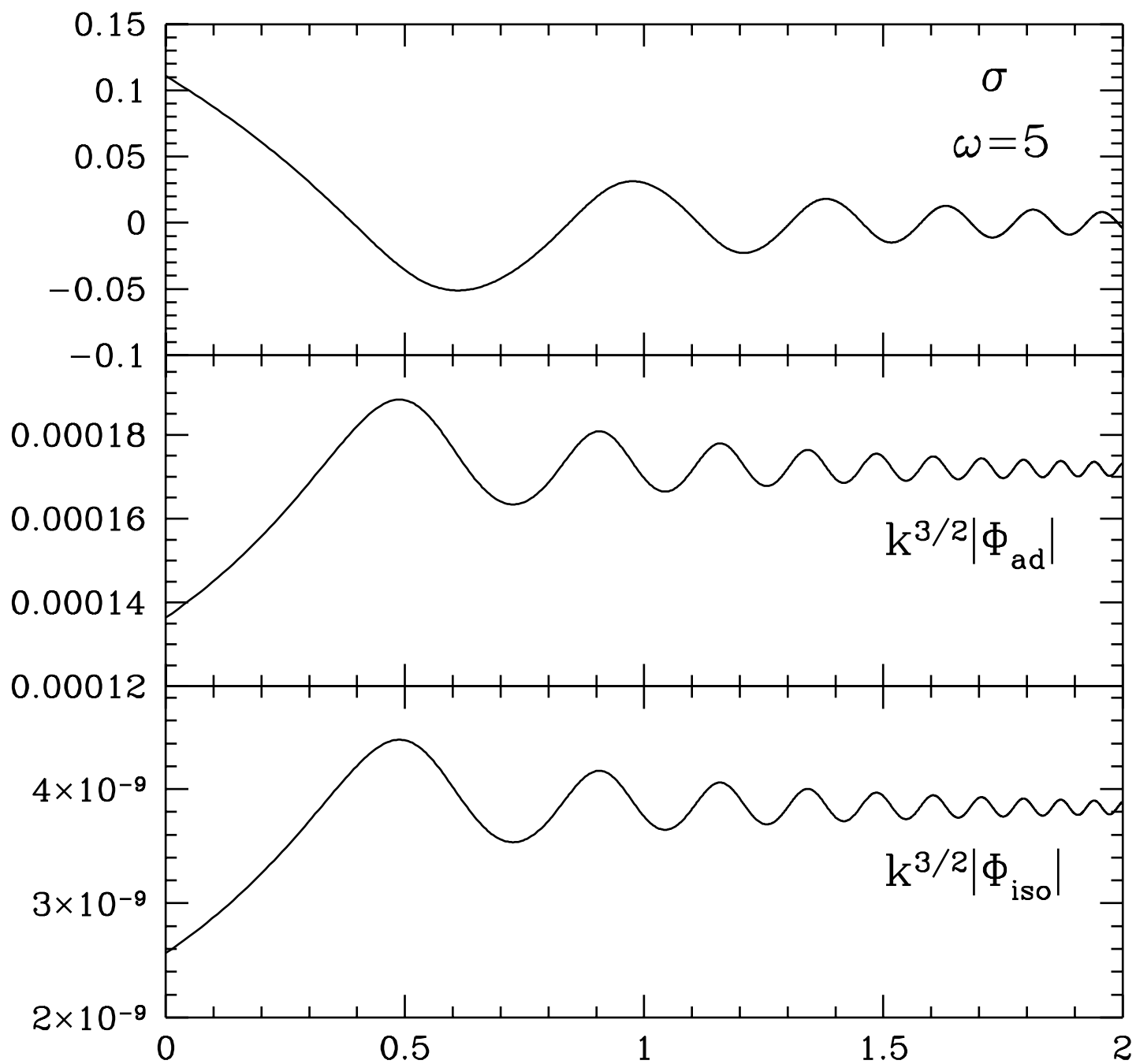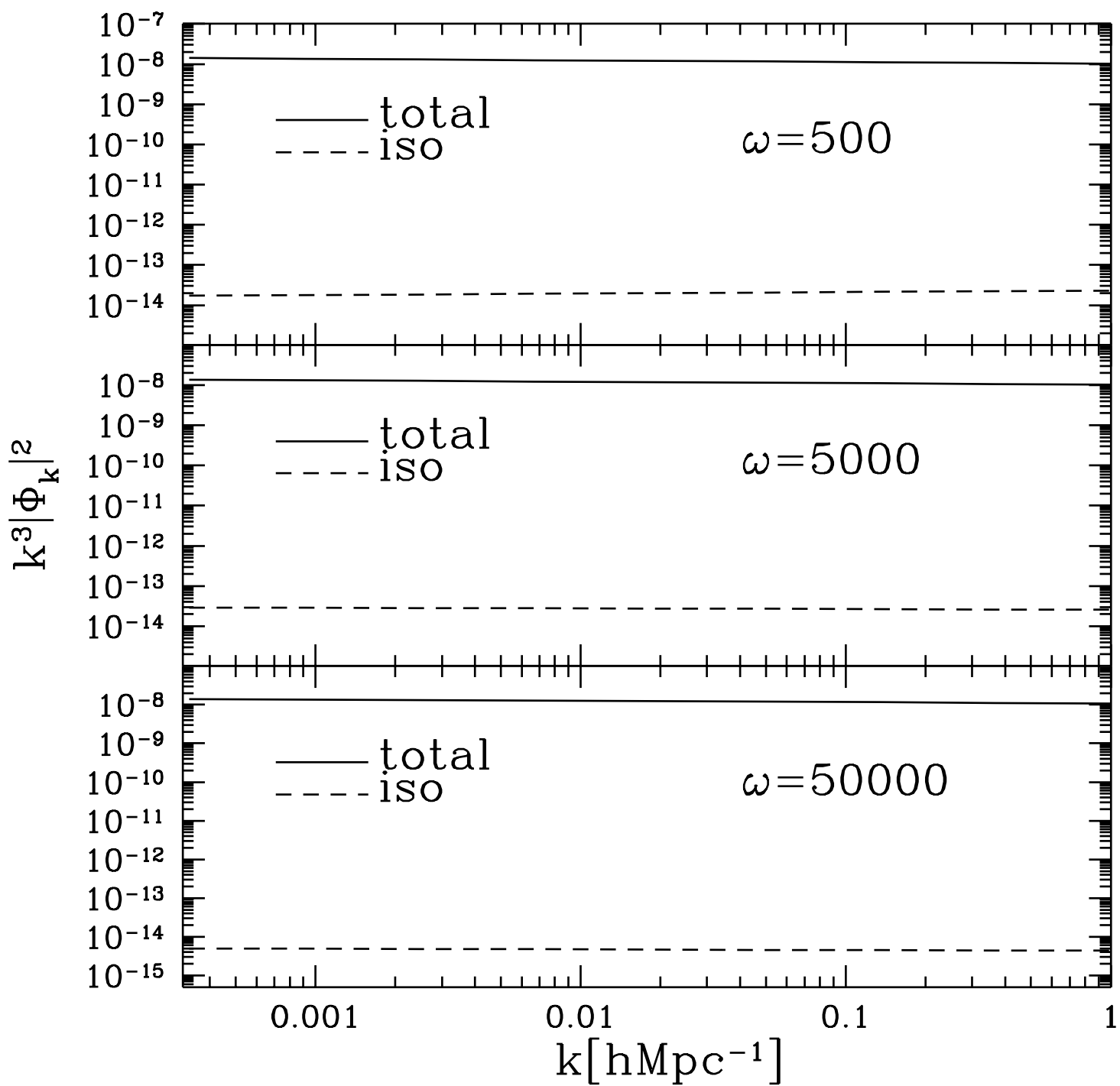
σ

$\omega=500$

$k^{3/2}|\Phi_{ad}|$

$k^{3/2}|\Phi_{iso}|$

$\ln R(t)/R_e$

Fig.1(a)

σ

$\omega=5$

$k^{3/2}|\Phi_{ad}|$

$k^{3/2}|\Phi_{iso}|$

$\ln R(t)/R_e$

Fig.1(b)

$k^3|\Phi_k|^2$

total
iso
$\omega=500$

10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}

total
iso
$\omega=5000$

10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}

total
iso
$\omega=50000$

10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}

0.001 0.01 0.1 1

$k[hMpc^{-1}]$

Fig.2

Fig.3

Fig.4